Appliances of First Degree and Second Degree Mathematical Equations in Solving of Some Physics Problems

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Abstract. *Mathematics is not only an abstract object. It had many appliances in Physics, Chemistry, Astronomy or Biology. I want to reveal that more problems in Physics are solved with mathematical equations of first and second degree.*
The classics unknowns x, y from Mathematics are substitutes in Physics by velocity v, time t, acceleration a. If you know to solve these equations on Mathematics, you must know how to apply them in others sciences too. You must be careful that in physics phenomena not exist negatives sizes (in Mathematics first degree and second degree equations can have positives and negatives solutions).

Keywords. Equations, Mathematics, Physics.

1. Introduction

For solving any problem of physics, you need know mathematics, like first degree and second degree equations. If you want to apply math formula in physics, classical unknowns x, y, $x_{1,2}$ and $y_{1,2}$ must be substituted with speed v, time t, distance x, l, d, or acceleration a.

From the first school years, we learned to measure time and distance. In expressions like “break between courses is ten minutes” or “after two hours I will meat my friends”, on observe that time measures represent only positive numbers. We’ll never hear someone who tell “- ten minutes” or “-two hours”. Also we say “the distance from home to school is 1 km” or “distance between two cities is 200 km”, but no “-1 km” or “- 200 km”.

When we drive the car, we see that the speed recorder begin from 0 to high speeds (300 km/h and over 300 km/h) if we have a race car!

We will never hear to distances by “-150 km”, for example.

All these examples give to us differences, between mathematics and physics, very important. A physics problem not must tackle like some mathematical problem. This means, among other things, that the values for physical measures can be only positive numbers. When we will find these values, we must add the unities for measure.

In Romania, the Mathematics curricula is very extent and the teacher, generally not have the time to accentuate the parallel between Mathematics and another sciences, so students know to solve equations from mathematic point, but they don’t understand very well the physical significance.

2. Examples

1.An object of mass m_1= 0,1kg is thrown upwards with a velocity of v_0= 15, 5 m/s. After t_1=1,2 s, it meet another body of mass m_2 =0,3 kg in free fall from H = 150m high. The second body was dropped after T = 0,5 s from the upwards thrown of the first body. Calculate:

a) the velocity v_1 of the first body after a time t_1;

b) the vertical distance h_1 of the first body after a time t_1;

c) the total energy of the first body when it meet the second body;

d) the vertical distance and the velocity of the second body at the meeting time;

e) the total energy of the second body at the meeting time.

Solution:

a) To lift an object to a high h above the ground, works has to be done against the force of gravity. The acceleration of the first body is a = -g, because the first body is thrown upwards and the acceleration of the second body is $a = g$, because the second body is in free fall. g is the gravitational field strength, $g = 9{,}8\ m/s^2$. Objects are represented in Fig. 1.

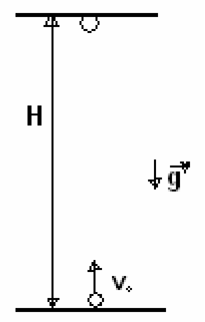

Figure 1. Objects for problem 1

The velocity law for the first body is:

$$v_1 = v_0 + a\, t_1 \quad (1)$$

and because the acceleration of the first body is $a = -g$,

$$v_1 = v_0 + (-g)\, t_1$$

$$v_1 = v_0 - g\, t_1\ .$$

The distance law is:

$$h_1 = h_0 + v_0 t_1 + \frac{a t_1^2}{2} \quad (2)$$

$$h_1 = h_0 + v_0 t_1 + \frac{-g t_1^2}{2}$$

$$h_1 = h_0 + v_0\ t_1 - \frac{g t_1^2}{2}\ ,$$

The Galileo's equation for the first body is:

$$v_1^2 = v_0^2 + 2ah_1 \quad (3)$$

$$v_1^2 = v_0^2 - 2gh_1$$

If use equation (3) and put in numbers, we have:

$$v_1^2 = 15 - 9{,}8 \times 1{,}2 = 15 - 11{,}76 = 3{,}24$$

$$v_1 = \sqrt{3{,}24} = \sqrt{\frac{324}{100}} = 1{,}8 m/s$$

b) Put in numbers equation (2) and known that $h_0 = 0$:

$$h_1 = 15{,}5 \times 1{,}2 - \frac{9{,}8 \times (1{,}2)^2}{2}$$

$$h_1 = 18{,}60 - \frac{9{,}8 \times 1{,}44}{2}$$

$$h_1 = 18{,}60 - \frac{14{,}112}{2}$$

$$h_1 = 18{,}60 - 7{,}556$$

$$h_1 = 10{,}044\ m.$$

c) When the first object meets the second object, it had both kinetic energy and gravitational potential energy.

Formula for kinetic energy first:

$$E_c = \frac{m_1 v_1^2}{2} \quad (4)$$

then numbers

$$E_c = \frac{0{,}1 \cdot 3{,}24}{2} = \frac{0{,}324}{2} = 0{,}162\ J$$

Formula for gravitational potential energy is

$$E_p = m_1 \cdot g \cdot h_1 \quad (5)$$

then numbers

$$E_p = 0{,}1 \cdot 9{,}8 \cdot 10{,}044 = 9{,}843\ J$$

Total mechanical energy for the first body is

$$E_t = E_c + E_p \quad (6)$$

and in numbers

$$E_t = 0{,}162\ J + 9{,}843\ J = 10{,}005\ J$$

d) The velocity of the second body at the meeting time is calculated using velocity law for it. Knowing that $v_0 = 0$ and it reached h_2 after $t_1 + T$:

$$v_2 = g\,(t_1 + T) \quad (7)$$

Put in numbers

$$v_2 = 9{,}8(1{,}2 + 0{,}5)$$

$$v_2 = 9{,}8 \times 1{,}7 = 16{,}66\ m/s$$

Distance travelled by the second mass at the meeting time is calculated with the Galileo's equation:

$$v_2^2 = v_0^2 + 2ah_2 \quad (8)$$

Knowing that $v_0 = 0$ and $a = g$, equation (8) is

$$v_2^2 = 2\, g\, h_2$$

$$h_2 = \frac{v_2^2}{2g}$$

$$h_2 = \frac{277{,}55}{19{,}6}$$

$$h_2 = 14{,}16 m.$$

This is the meeting high.

e) For the total energy of the second object at the meeting time we must calculate the kinetic energy and the gravitational energy for him and use equation (6):

$E_{c2} = \frac{m_2 \cdot v_2^{\ 2}}{2} = \frac{0,3 \cdot 277,55}{2} =$ 41,632 J

$E_{p_2} = m_2 \cdot g \cdot (H - h_2) =$ 0,3 9,8 (150 – 14,16) = 399,36 J

E_t = $E_{c\,2}$ + $E_{p\,2}$ = 41,632 J + 399,36 J = 440,992 J

2. An object with mass m_1 = 2 kg, descended on an inclined plane with α = 30 ^{0}, from a vertical height h = 3 m. His initial velocity is v_{0_1} =15 m/s and the friction coefficient is μ = 0,1. After a distance x = 4m, to a horizontal plane, it meet another object m_2 = 1 kg, who is at rest. Between them is an inelastic collision. On horizontal plane the friction coefficient is μ = 0,1 too.

Calculate:

a)Slope's length and first object velocity v_1, at the base of the slope ;

b)Velocity v_2, of the first object before collision ;

c)Velocity v of both objects, after collision.

Solution:

Objects are represented in Fig. 2.

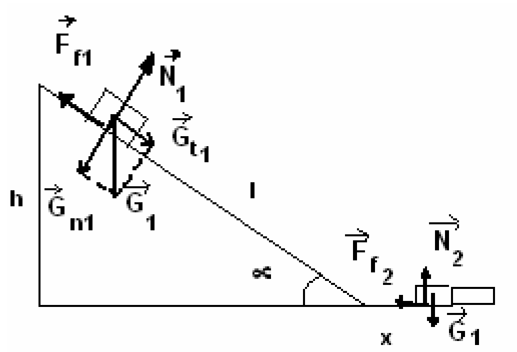

Figure 2. Objects for problem 2

a) Knowing the high from which the first object is going down, h = 3 m, as well the fact that the inclined plane is a rectangular triangle with α=30 ^{0}, we can use the formula

$$\sin \alpha = \frac{h}{l} \Rightarrow l = h \sin \alpha \qquad (9)$$

In numbers:

l = 3 sin 30 ^{0}

$l = 3 \times \frac{1}{2}$

$l = 3 \times 1,5$

l= 4,5 m who is inclined plane's length.

From Newton's Second Law of motion

$$\vec{G}_1 + \vec{N}_1 + \vec{F}_{f\,1} = m_1 \vec{a}_1 \qquad (10)$$

Ox: $G_{t1} - F_{f1} = m_1 \cdot a_1$ (11)

Oy: $N_1 - G_{n1} = 0$ (12)

The second member in (12) equation is 0 because the object has no component of acceleration to Oy axis.

But we know that

$G_{t1} = G_1 \sin \alpha$ (13)

$F_{f1} = \mu N_1$ (14)

$G_{n1} = G_1 \cos \alpha$ (15)

$G_1 = m_1 g$ (16)

With equations (11) - (16), we obtain

$m_1 g \sin \alpha - \mu m_1 g \cos \alpha = m_1 a_1$ (17)

$a_1 = g(\sin \alpha - \mu \cos \alpha)$ (18)

In numbers

$a_1 = 9,8(\sin 30 + 0,1 \cos 30)$

$a_1 = 9,8 \cdot \left(\frac{1}{2} + 0,1 \frac{\sqrt{3}}{2} \right)$

$a_1 = 9,8 \cdot \left(0,5 + 0,1 \cdot \frac{1,73}{2} \right)$

$a_1 = 9,8 \cdot (0,5 + 0,1 \cdot 0,86)$

$a_1 = 5,74 m/s^2$

This equation represents object's acceleration. Also we know his initial velocity and Galileo's equation

$$v_1^{\ 2} = v_{0_1}^{\ 2} + 2 \cdot a_1 \cdot l \qquad (19)$$

In numbers

$v_1^{\ 2} = 15^2 + 2 \times 5,74 \times 4,5 = 276,66$

$v_{1,2} = \pm\sqrt{276,66} = \pm 16,63$.

As we know, in Physics negatives measures doesn't exist, so the solution is v_1=16,63 m/s.

b)To calculate velocity v_2 of the first object before collision, we must use Newton's Second Law of motion, that's mean equation (10), to find out its acceleration a_2 on the horizontal plane. Also equations (11) and (12) becomes

Ox: $-F_{f_2} = m_1 \cdot a_2$ (20)

Oy : $N_2 - G_1 = 0$ (21)

or $N_2 - m_1 g = 0$

so from (20)

$-\mu\, m_1 g = m_1 \cdot a_2$

$a_2 = -\mu\, g$ (22)

Galileo's equation on the horizontal plane is

$$v_2^{\ 2} = v_1^{\ 2} + 2 \cdot a_2 \cdot x$$

$$v_2^{\ 2} = v_1^{\ 2} - 2\mu g x$$

In numbers

$v_2^2=276,66-2\times 0,1\times 9,8\times 4$

$v_2^2=268,82$

$$v_{2_{1,2}} = \pm\sqrt{268,82}$$

$$v_2 = 16,39 m/s$$

From same reasons like point a) we choose the positive solution with physics significance.

c)To find out velocity v of both objects, after collision we use The Principle of Conservation of Momentum in an inelastic collision:

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2)v \qquad (23)$$

But the second object is at rest, so $v_2=0$. Equation (23) becomes

$$m_1 v_1 + 0 = (m_1 + m_2)v \qquad (24)$$

$$v = \frac{m_1 v_1}{m_1 + m_2} \qquad (25)$$

In numbers

$$v = \frac{2\times 16,39}{2+1}$$

In conclusion, the velocity v of both objects, after collision, is

$$v = 10,92 m/s$$

3. Conclusions

Like we know, the movement of natural objects is a complex movement. In these problems I wanted to reveal that is not enough to know only some chapters of Physics, but to resolve problems of synthesis. When we will finish resolving the problem, by mathematic point, we must to put in evidence the pure physical sense, in that context. The "Hands on Science" network provides a frame to improve in-school scientific education.

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